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Comparison of Nonlinear and Linear Multireceiver Detection Systems

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Summary—Detection of a binary transmission by both optimum and suboptimum nonlinear and linear multireceivers is considered by comparing their asymptotic performance characteristics. The multichannel model is presumed to be of the Rician type. Particularly, we consider Turin's nonlinear specular-coherent multireceiver and the nonlinear noncoherent Pierce-Stein multireceiver. These two termination error rate characteristics are graphically compared for low and high output signal-to-noise ratios. The performance characteristics of two other coherent linear multireceivers, one optimum and one easier implemented suboptimum, are derived and compared with the above-mentioned nonlinear multireceivers. The numerical results indicate system design trends and provide information on the degradation or improvement afforded by employing nonlinear detection systems as compared with linear detection systems.

In particular, the optimum nonlinear coherent multireceiver is difficult to implement. It is shown that, for multichannels which are largely specular in nature, a more easily implemented linear coherent unit behaves optimally for all practical purposes. For channels which are largely scatter in nature it is shown that the linearized suboptimum system performance is highly inferior to the corresponding optimum coherent unit. In these situations, the noncoherent "square-law combining" system would be more reliable than the suboptimum coherent unit. In fact, for large scatter components we find that the noncoherent unit performs almost identically to the nonlinear coherent unit. This is due to the signal suppression effects known to occur in all nonlinear detectors throughout the field of statistical detection theory.

Introduction

EXACT AND ASYMPTOTIC results have been given[1,2] for the error probability of coherent and noncoherent multireceivers which are presumed to be connected to the Rician fading multichannel. The situation depicts, in some sense, both diversity communications, planetary relay communications, and resolvable multipath communications. The purpose of the present paper is to establish the asymptotic performance characteristics for Turin's nonlinear coherent multireceiver,[3] and to compare these results with those obtained by this author[1] and with another more easily implemented suboptimum linear system.

In the system under consideration each of M channels is presumed to be perturbed by additive white Gaussian noise of spectral density $N_0 w/\mathrm{cps}$ single-sided and by the slow Rician fading phenomena. The noises in each link are assumed to be mutually independent. During a signaling interval of T seconds one of two equal-energy equiprobable orthogonal waveforms is transmitted into the set of channels. The binary signal is first disturbed by the fading characteristic and then superposed with additive noise.

The nonlinear coherent multireceiver shown in Fig. 1 passes each arriving waveform into a pair of filters matched to the transmitted waveforms and follow-up square-law envelope detectors. For this termination the switch positions are as indicated in Fig. 1. In making a binary decision the outputs of the matched filters and envelope detectors are sampled at the conclusion of the signaling interval, properly weighted and summed.[1] The larger sum determines the more likely transmitted signal. We shall derive the asymptotic performance characteristics for multichannels with both large and small distinct specular components and compare these results with the asymptotic performance of three other multireceiver units. In what follows we refer to the four multireceivers as the Pierce-Stein square-law multireceiver, the Turin specular-coherent multireceiver and the optimum and suboptimum linear coherent multireceiver.

Briefly, the decision for the Pierce-Stein unit is made

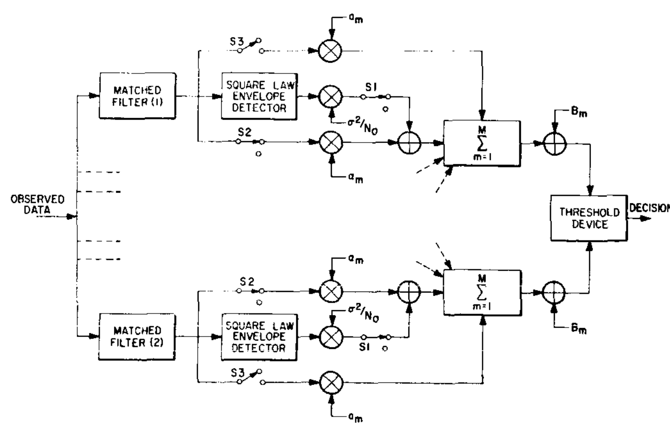

Fig. 1—Detection system (mth branch).

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[1] W. C. Lindsey, "Asymptotic performance characteristics for the coherent multireceiver and noncoherent multireceiver operating through the Rician fading multichannel," IEEE TRANS. ON COMMUNICATION AND ELECTRONICS, vol. 82, pp. 755–761; January, 1964. Also Jet Propulsion Lab., Pasadena, Calif. Tech. Rept. 32-440; 1963.

[2] ——, "Error Probabilities for Rician Fading Reception of Binary and N-ary Signaling," Jet Propulsion Lab., Pasadena, Calif., Tech. Rept. 32-450; 1963. Much of the notation for the present paper is derived from this report and Lindsey.[1]

[3] G. L. Turin, "Communication through noisy, random-multipath channels," 1956 IRE CONVENTION RECORD, pt. 4, pp. 154–165.

by comparing the sums of samples taken from the outputs of the square-law devices. (Open S_2 in Fig. 1.) The outputs from the matched filter are neglected in this termination. For the optimum linear coherent multireceiver the decision is made by comparing the summed-weighted samples taken in proper phase from the matched filter outputs.[4] This termination is depicted by the opening of switches S_1 and S_2 and the closing of S_3. The outputs of the square-law devices are neglected in this termination. In the suboptimum linear system we merely open switches S_1 and S_3. This system is the linearized version of the specular-coherent Turin unit.[5]

Asymptotic Analysis

The error probability for the specular-coherent multireceiver is a rather complicated function of four communication parameters: M, the number of channels; ρ_i, the SNR of the ith channel specular component; β, the SNR of the random channel component; and γ_i^2, the ratio of the ith fixed channel component to the mean-squared value of the random component.

The probability density function governing the summed signal plus noise and noise samples may be shown[7] to be given, respectively, by

$$p_2(Y) = \frac{Y}{c}\left[\frac{Y}{A}\right]^{M-1} \exp\left[-\frac{Y^2 + A^2}{2c}\right] I_{M-1}\left[\frac{A}{c}Y\right]; \; Y \geq 0$$
$$= 0 \text{ elsewhere} \tag{1}$$
$$p_1(X) = X\left[\frac{X}{B}\right]^{M-1} \exp\left[-\frac{X^2 + B^2}{2}\right] I_{M-1}[BX]; \quad X \geq 0$$
$$= 0 \text{ elsewhere}$$

where

$$B^2 = \frac{2}{\beta}\sum_{m=1}^{M}\gamma_m^2 = \frac{2}{\beta^2}\sum_{m=1}^{M}\rho_m$$
$$A^2 = (1+\beta)^2 B^2, \qquad c = 1 + \beta. \tag{2}$$

A decision is in error if the variate $(Y - X)$ chances to be negative. In fact, the probability of this occurring has been found;[7] *i.e.*, $\int_0^\infty p, (Y)\, dY \int_X^\infty p, (X)\, dX$. In the present paper, however, we are not interested in the exact result.

Consider now the multichannel model which possesses small specular components. The error rate for this situation is, in statistical parlance, given by the ratio of two χ-squared variates modified by exponential factors. This ratio is well known and turns out to be, for large β, asymptotic to

$$_{sc}P_E(M) \sim \frac{1}{2}\binom{2M}{M}\left[\frac{1}{\beta}\right]^M \prod_{m=1}^{M} \exp\left[-\gamma_m^2\left(1 + \frac{2}{\beta}\right)\right]$$
$$_{sc}P_E(M) \sim \frac{1}{2}\binom{2M}{M}\left[\frac{1}{\beta}\right]^M \prod_{m=1}^{M} \exp\left[-\rho_m\left(\frac{2+\beta}{\beta^2}\right)\right] \tag{3}$$

where $\binom{a}{b}$ is the binomial coefficient given by $a!/b!(a-b)!$.

If we now consider the case where the multichannel possesses large specular components we may argue that the decision variate $D = Y - X$ becomes Gaussian in two ways. On the one hand, when the number of channels M is large, the Central Limit Theorem may be invoked to establish the normality. On the other hand, one may resort to letting $M = 1$ in (2), and use the asymptotic expansion of $I_0(z)$ to show that the individual combined samples become Gaussian. As a consequence the summed samples are Gaussian. Using Weber's first exponential integral[8] it is easy to show that the ith moment of variate Y is given by

$$m_i(Y) = (2c)^i \frac{\Gamma(M+i)}{\Gamma(M)} F\left(-i, M; -\frac{A^2}{2c}\right) \tag{4}$$

where $F(a, b; x)$ is the hypergeometric function.[8] Thus the mean and variance of Y are found from (4) to be

$$m_1(Y) = 2cM + A^2$$
$$\sigma^2(Y) = 4cA^2 + 4c^2M. \tag{5}$$

Similar results may be found for the variate X by letting $c = 1$ and replacing A by B. The output SNR for the summed samples is given by

$$SNR = \frac{[A^2 + 2Mc - B^2 - 2M]^2}{4c^2M + 4M + 4cA^2 + 4B^2} \tag{6}$$

which becomes, by use of (2),

$$SNR = \frac{[B^2(c^2-1) + 2\beta M]^2}{4B^2(c^3+1) + 4M(c^2+1)}$$
$$= \frac{\left[2\sum_{m=1}^{M}\rho_m(2+\beta) + 2\beta M\right]^2}{8[1+(1+\beta)^3]\sum_{m=1}^{M}\rho_m + 4M\beta^2[1+(1+\beta)^2]} \tag{7}$$
$$SNR \sim \frac{(2+\beta)^2}{2[1+(1+\beta)^3]}\sum_{m=1}^{M}\rho_m$$

where we have assumed $\gamma_m^2 > 1$.

Assuming that the decision quantity is Gaussian, the approximation to the multireceiver error probability is

$$_{sc}P_E(M) = \frac{1}{2}\left[1 - \sqrt{\frac{2}{\pi}}\int_0^{\sqrt{SNR}} \exp\left[-\frac{z^2}{2}\right] dz\right], \tag{8}$$

[4] In this termination we presume that the multireceiver knows *a priori*, or through measurement, the multichannel gain characteristic.

[5] The optimality of the multireceiver units considered in this paper is discussed by Lindsey[2,6] and Turin.[3]

[6] W. C. Lindsey, "A Wideband Adaptive Communication System," Ph.D. dissertation, Purdue University, Lafayette, Ind.; 1962. See under title J. C. Hancock and W. C. Lindsey, Purdue University, Lafayette, Ind., vol. III, AF Contract 33(616)-8283, PRF 2906.

[7] W. C. Lindsey, "Performance Analysis for the Coherent Multireceiver in Specular and Random Channels," Space Programs Summary, Jet Propulsion Lab., Pasadena, Calif., No. 37-21, vol. IV; April, 1963.

[8] G. N. Watson, "Theory of Bessel Functions," Cambridge University Press, Cambridge, England; 1958.

or, asymptotically, we have

$$_{sc}P_E(M) \sim \frac{1}{\sqrt{2\pi(SNR)}} \exp\left[-\frac{SNR}{2}\right]. \tag{9}$$

At $\beta = 0$, (7) checks as it should.

We have shown[1] that the asymptotic performance characteristic for the linear coherent multireceiver is given by

$$_cP_E(M) \sim \frac{1}{2}\binom{2M}{M}\left[\frac{1}{2\beta}\right]^M \prod_{m=1}^{M} \exp\left[-\gamma_m^2\right] \tag{10}$$

for $\lambda = 0$ and $\beta > 1$. For the Pierce-Stein noncoherent multireceiver we have[1]

$$_{nc}P_E(M) \sim \frac{1}{2}\binom{2M}{M}\left[\frac{1}{\beta}\right]^M \prod_{m=1}^{M} \exp\left[-\gamma_m^2\right]. \tag{11}$$

Denoting then the error rate for the specular-coherent termination by $_{sc}P_E(M)$ we may conveniently compare the individual receiver structures for multichannels which support small specular components.

Taking the ratio of (10) to (3) we have

$$R_1(M) = \frac{_cP_E(M)}{_{sc}P_E(M)} = [\tfrac{1}{2}]^M \prod_{m=1}^{M} \exp\left[+\frac{2\gamma_m^2}{\beta}\right]. \tag{12}$$

While the ratio of (3) to (11) gives

$$R_2(M) = \frac{_{sc}P_E(M)}{_{nc}P_E(M)} = \prod_{m=1}^{M} \exp\left[-\frac{2\gamma_m^2}{\beta}\right]. \tag{13}$$

Further assuming all subchannel specular components are equally reliable, (12) and (13) become

$$\begin{aligned} R_1(M) &= (\tfrac{1}{2})^M \exp\left[\frac{2M\gamma^2}{\beta}\right] = (\tfrac{1}{2})^M \exp\left[\frac{2M\rho}{\beta^2}\right] \\ R_2(M) &= \exp\left[-\frac{2M\gamma^2}{\beta}\right] = (\tfrac{1}{2})^M \exp\left[-\frac{2M\rho}{\beta^2}\right]. \end{aligned} \tag{14}$$

Finally, we consider the multireceiver which neglects using the samples taken from the output of the envelope detectors; *i.e.*, the decision in this case is made by summing the nonweighted samples taken from the output of the set of matched filters. This is the linearized version of Turin's nonlinear multireceiver. Note the simplicity of implementing this unit as compared with the nonlinear unit. For this situation the signal plus noise statistics for the mth sample are given by

$$p(y_m) = \frac{1}{\sqrt{2\pi c}} \exp\left[-\frac{(y_m - \sqrt{\rho_m})^2}{2c}\right] \tag{15}$$

for the filter output which happens to be matched to the transmitted waveform[9] and

$$p(x_m) = \frac{1}{\sqrt{2\pi c}} \exp\left[-\frac{x_m^2}{2c}\right] \tag{16}$$

for the filter output which is presumed matched to the empty channel. The density functions for the decision variates $X = \sum_{m=1}^{M} x_m$ and $Y = \sum_{m=1}^{M} y_m$ are

$$\begin{aligned} p(Y) &= \frac{1}{\sqrt{2\pi cM}} \exp\left[-\frac{\left(Y - \sqrt{\sum_{m=1}^{M} \rho_m}\right)^2}{2cM}\right] \\ p(X) &= \frac{1}{\sqrt{2\pi cM}} \exp\left[-\frac{X^2}{2cM}\right] \end{aligned} \tag{17}$$

where $c = 1 + \beta$.[10] The error rate, under the conditions imposed, is easily found to be

$$P_E(M) = \frac{1}{2}\left\{1 - \operatorname{erf}\left[\sqrt{\sum_{m=1}^{M} \frac{\rho_m}{1+\beta}}\right]\right\} \tag{18}$$

where

$$\operatorname{erf}(z) = \sqrt{\frac{2}{\pi}} \int_0^z \exp\left[-\frac{t^2}{2}\right] dt.$$

We have, asymptotically, for large ρ_m and small β

$$P_E(M) \sim \frac{1}{\sqrt{2\pi \sum_{m=1}^{M} \frac{\rho_m}{1+\beta}}} \prod_{m=1}^{M} \exp\left[-\frac{\rho_m}{2(1+\beta)}\right], \tag{19}$$

while for large β and small ρ_m we obtain

$$P_E(M) \sim \frac{1}{2}\left[1 - \sqrt{\frac{2}{\pi}} \sum_{m=1}^{M} \frac{\rho_m}{1+\beta}\right]. \tag{20}$$

Notice that in the absence of noise the probability of error becomes

$$P_E(M) = \left\{\tfrac{1}{2}1 - \operatorname{erf}\left[\sqrt{\sum_{m=1}^{M} \gamma_m^2}\right]\right\}. \tag{21}$$

This shows that the decision may still be in error and that, in fact, the error rate depends on the strength of the specular or fixed component to the mean-squared strength of the random component.

System Comparison

In order to obtain knowledge about the effectiveness of the nonlinear detection system characteristic, we compare the asymptotic performance with that of the linear detector. We presume, in this discussion, that all subchannel specular components are equally reliable. Fig. 2 shows a graphical plot of (12) and (13) taken in appropriate regions of the ratio $\gamma^2/\beta = \rho/\beta^2$. Consider, first, the comparison of the Pierce-Stein combiner with that of the optimum specular-coherent combiner. This comparison is given by $R_2(M)$ in Fig. 2. As one might suspect, we find that for very small values of γ^2/β the two detection system

[9] This result appears in unpublished work of the author entitled "Probability Distributins for Practical Applications." A copy may be obtained by writing the author.

[10] The analysis may be carried out, in this case, for the multichannel model which possesses scatter components of distinct mean strength $2\sigma_m^2$ for all $m = 1, 2, \cdots M$.

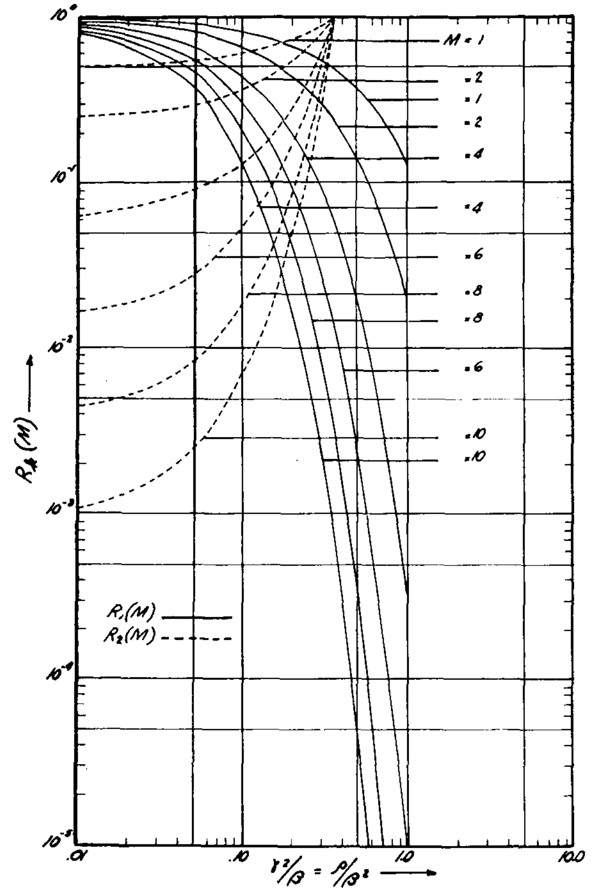

Fig. 2.

characteristics approach each other; this is true because the specular-coherent multireceiver makes its decision based approximately on the summed square-law detector outputs. For large values of the ratio γ^2/β we see that the samples taken from the matched filter outputs in combination with those from the envelope detector are more important, *i.e.*, the establishment of coherency does indeed improve the performance. In terms of equipment implementation it would, of course, be more difficult to build the specular-coherent multireceiver. The decision as to whether the improvement obtained is worth additional amplitude and phase measuring equipment is debatable. In fact, in order for the design engineer to choose the noncoherent over the coherent termination requires further *a priori* knowledge about the channel involved, system application, the detection reliability required, and the complexity of equipment which the system establishment may afford. For channels which support rather small specular components the noncoherent termination would be the most economical and would perform approximately as well as the more sophisticated coherent termination.[11]

Turning now to the comparison of the nonlinear multireceiver with that of the linear multireceiver, we find that for small values of γ^2/β, $R_1(M)$ is less than one. (See Fig. 2.) This shows that for this region of the parameter γ^2/β the nonlinear specular-coherent multireceiver performs inferiorly to the linear coherent multireceiver. The reason for such performance, even though both receivers are terminated coherently, is easily explained when one recalls the familiar signal suppression effects common in all nonlinear detectors. Note that the asymptotic approximation begins to break down for values of $\rho/\beta^2 > 0.4$ or regions where $\beta > \sqrt{2.5\rho}$.

Finally, we compare the asymptotic performance characteristics when ρ is large and β is small. To do this we need a result derived in Lindsey[1] which is to be valid for this region of comparison. We found the asymptotic value to be given by

$$ {}_cP_E(M) \sim \left[\frac{2}{2+\beta}\right]^{M-1} \frac{\exp\left[-\dfrac{M\rho}{2+\beta}\right]}{\sqrt{2\pi M\rho}} \tag{22} $$

where we have assumed $\gamma_m^2 = \gamma^2$ for all $m = 1, \cdots M$. Taking the ratio of (22) to (9) we find

$$ \frac{{}_cP_E(M)}{P(k)} = G_k(M) = \left[\frac{2}{2+\beta}\right]^{M-1} \sqrt{\frac{(SNR)_k}{M\gamma^2\beta}} \cdot \left[\exp\left\{\frac{(SNR)_k}{2} - \frac{M\gamma^2\beta}{2+\beta}\right\}\right] \tag{23} $$

where k signifies which coherent multireceivers are being compared. If $k = 1$ we compare the suboptimum coherent multireceiver with the optimum coherent multireceiver. In this case

$$ (SNR)_1 = \frac{\gamma^2\beta}{2(1+\beta)} $$

whereas $k = 2$ denotes the comparison of the coherent multireceiver with that of the specular coherent multireceiver. In this case $(SNR)_2$ is given by (7). Graphically illustrated in Fig. 3 is a plot of (23) for several values of the communication parameters M, β, γ^2 and $k = 1, 2$. The value of γ^2 is chosen so as to typify a multichannel model which is largely specular in nature. This, for example, could be a planetary relay link, a passive earth-satellite system, or a deep-space probe telemetry system. Comparing the curve $G_1(M)$ with $G_2(M)$, for any $M = 1$, 2, 4, 6, 8 and 10 indicates that the optimum nonlinear multireceiver is slightly superior to the suboptimum linear multireceiver. Since this optimum multireceiver is more difficult to implement one may conclude that the more easily implemented suboptimum multireceiver behaves optimally, for all practical purposes. We reiterate that we are speaking about channels which are largely specular in nature. Comparing the optimum channel measuring

[11] System behavior for large noise is also of interest. Turin[3] has shown that, for a large noise spectral density N_0 the information transferred through the channel is conveyed exclusively by the specular components.

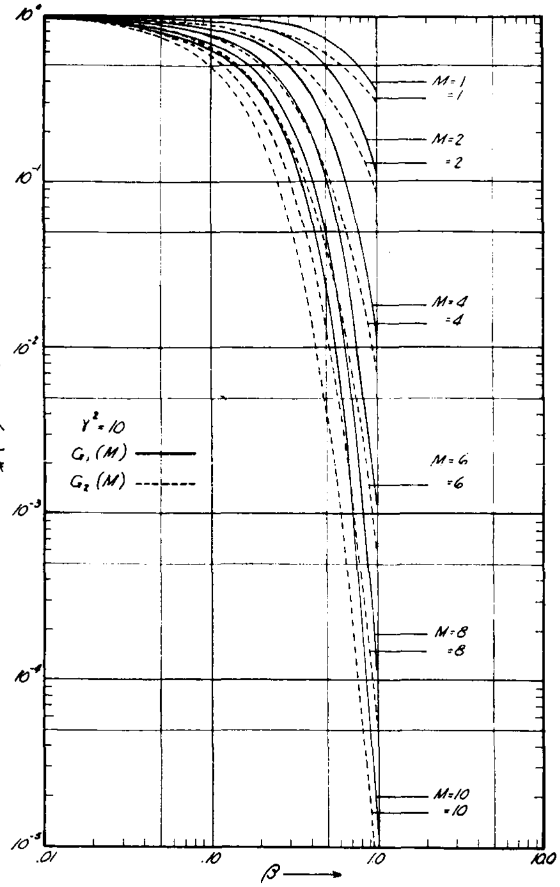

Fig. 3.

coherent multireceiver with either of the other two coherent units we find that the increase in performance obtained by employing dual reception is very minor. This is especially true, for all M, where the multichannel model approaches the specular case, *i.e.*, $\beta \sim 0$. For $M > 2$, however, the coherent unit yields a marked improvement. Particularly for $\beta > .2$, the coherent unit becomes superior approximately exponentially.

Conclusions

Asymptotic performance characteristics for optimum and suboptimum binary data-handling systems have been presented both analytically and numerically. The following are major conclusions of this study. The optimum nonlinear coherent receiver performance is slightly superior to that of a more easily implemented suboptimum linearized version of the same receiver. This presumes that the multichannel is largely specular in nature. For multichannels which are largely random (small specular components), noncoherent reception is for all practical purposes equivalent to specular-coherent reception. Finally, optimum linear detection with a receiver which is informed of the channel gain and phase characteristic by channel measurement is impreceptibly superior to the optimum nonlinear detector which has no *a priori* knowledge about the channel gain. This presumes a multichannel which acts largely as a specular reflector.

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